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A ray tracing method for evaluating the radiative heat transfer in porous media

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Abstract—A general ray-tracing method is proposed to determine the radiative properties of porous media. The particular case of random packing of equal sized spheres is studied. The method is used to calculate the forward and backward fluxes inside the medium. The two-flux model is then used to derive the absorption and scattering coefficients of the medium, from which the radiant conductivity can be calculated. Quantitative agreement is obtained in comparison with previous experimental and numerical works.

INTRODUCTION

This paper describes a general method to determine the radiative properties of porous media, with special attention to the random packing of spheres through densification. The packings of spheres are of great importance in modern technology in such diverse domains as high-performance cryogenic insulation, coal combustors, chemical reactors, nuclear fuel rods and powder metallurgy.

Considering that the medium is evacuated, two transport mechanisms are relevant: conduction through the solid phase and radiative transfer through the voids. However, as long as the medium can be considered as optically thick, these mechanisms can be decoupled and considered separately [1, 2]. The authors proposed a model for the solid phase contribution [3], which is often preponderant. However, radiative transfer is strongly dependent on temperature, and some processes involve temperatures high enough for this mechanism to be significant. The formal problem is to determine the radiative conductivity of the medium. This conductivity, which shall be used in the thermal diffusion equation, should take into account the complex interactions of the radiation with the particulate material.

Several previous studies, most of them experimental, were devoted to determining radiative properties of sphere packings. For a complete review one may refer to Tien [4] or Tien and Vafai [5]. The experimental determination of the radiative flux intensity involves lots of difficulties due to the inaccurate measurement of the transmitted intensity, which is very weak. Thin layer samples allow direct transmission, which invalidates the use of the diffusion approximation. Therefore, computer simulations using the ray-tracing method, as described in a coming section, can be very useful since the here mentioned problems are eliminated. Such simulations were performed by Yang *et al.* [6] for random packings of uniform spheres. Yang *et al.* calculated the transmitted intensity for various packings and obtained a relation of this intensity with the packing height. The method proposed in this paper, which is close to the one presented by Yang *et al.*, is more consistent in the sense that the fluxes inside the packing are directly accessed and used to determine the radiative properties.

As the ray tracing method is used, the applicability of this work is restricted to media where the typical microscopic dimension is large compared to the wavelength of the incident radiation. The method can effectively take into account the dependent scattering, since it considers the multiple reflections occurring between the particles. The particles are supposed to be opaque and there are no near-field effects, which are valid assumptions for large metallic particles.

In this work, the medium under consideration is composed of equal sized spheres in a random packed bed. However, the method proposed can be used with more general porous media. The packing density is varied and the relation between the radiative conductivity and the relative density is obtained. The relative density is defined as the volumetric fraction occupied by the particles.

THEORETICAL BASIS—THE TWO-FLUX MODEL

The equations leading to the two-flux model were first proposed by Schuster [7] to describe the transmission of light through fog. These equations were later adapted by Hamaker [8] who extended the theory to include thermal radiation and the combined effect

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NOMENCLATURE

- a absorption coefficient
- C_1, C_2 arbitrary constants
- E dimensionless radiative exchange factor
- i radiative flux
- I ray energy bundle
- k_r radiative conductivity
- R porous medium characteristic dimension, particle radius
- s scattering coefficient
- T absolute temperature
- z coordinate in flow direction.

Greek symbols
- α radiation parameter, $\alpha = (a(a+2s))^{1/2}$
- β $2\pi R/\lambda$, size parameter
- ε emissivity
- λ incident radiation wavelength
- σ Stefan–Boltzmann constant.

Superscripts
- $+$ forward
- $-$ backward.

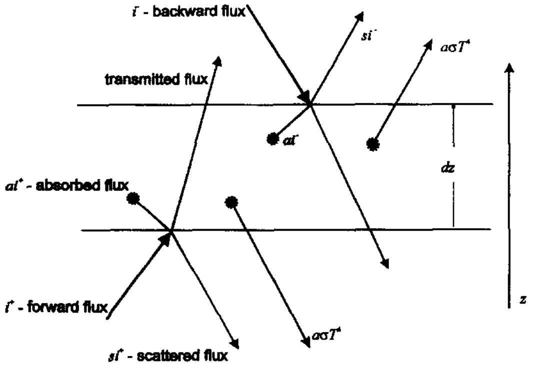

Fig. 1. Energy balance for a slab of semi-transparent medium under one-dimensional radiative flow.

of radiation and heat conduction. Consider a slab of a semi-transparent medium at absolute temperature T and a one-dimensional flux as in Fig. 1. The following two differential equations express the steady-state energy balance of the slab:

$$\frac{di^+}{dz} = -(a+s)i^+ + si^- + a\sigma T^4$$

$$\frac{di^-}{dz} = -(a+s)i^- + si^+ + a\sigma T^4, \qquad (1)$$

where i^+ and i^- are the forward and backward fluxes, σ is the Stefan–Boltzmann constant and T is the absolute temperature of the medium. The absorption and scattering coefficients, a and s, are defined as the fractions of energy of the propagating wave lost, respectively, by absorption and scattering per unit length in the z-direction. The terms on the right-hand side of the first equation of the system (1) are, respectively, the decrease of the forward flux by absorption and scattering, the increase due to scattering of the backward flux into the forward direction and the emission from the slab into the forward direction. The radiative conductivity is defined as

$$k_r = -\frac{i^+ - i^-}{\frac{dT}{dz}}. \qquad (2)$$

Assuming that spatial temperature variations are small, Hamaker [8] developed a solution to equation (1) by approximating T^4 by the first two terms of a Taylor's series expansion with respect to z coordinate. Let us assume that the medium is optically thick, which means that all incident radiation travels only a short distance before interacting with the medium. Hamaker [8] and Chen and Churchill [1] have shown that the radiative heat transfer can be treated as a diffusion process, so that the radiative conductivity can be calculated as

$$k_r = \frac{8\sigma T_m^3}{a+2s}, \qquad (3)$$

where T_m is the mean temperature of the medium used for the Taylor's expansion. Therefore, the thermal conductivity of the medium can be obtained from the absorption and scattering coefficients. As it will be shown in a coming section, these coefficients depend on the geometry of the packing and particles and on the emissivity of the particles surface. Consequently, they do not depend on the temperature, assuming that the emissivity is independent of the temperature. Since a and s are expected to be proportional to the projected area of the particles per unit volume of the medium, it is useful to define a dimensionless radiative exchange factor as

$$E = \frac{1}{R(a+2s)} \qquad (4)$$

where R is the average particle radius, or more generally the characteristic length of the porous medium. The radiative conductivity becomes

$$k_r = 8ER\sigma T_m^3. \qquad (5)$$

Consider now equations (1) applied to a low temperature light scattering problem, where the term $a\sigma T^4$ is negligible compared to the others. The solution of this system is

$$i^+(z) = C_1 e^{\alpha z} + C_2 e^{-\alpha z}$$

$$i^-(z) = C_1 \left(\frac{a+s+\alpha}{s}\right) e^{\alpha z} + C_2 \left(\frac{a+s-\alpha}{s}\right) e^{-\alpha z}, \quad (6)$$

where $\alpha = \sqrt{a(a+2s)}$. The constants C_1 and C_2 are obtained from the following boundary conditions:

$$i^+(0) = 1, \text{normalized input flow}$$

$$i^+(\infty) = i^-(\infty) = 0. \quad (7)$$

Finally

$$i^+(z) = e^{-\alpha z}$$

$$i^-(z) = \frac{a+s-\alpha}{s} e^{-\alpha z}. \quad (8)$$

With this set of equations, the coefficients a and s can be easily deduced from data on i^+ and i^-. Notice from equation (8) that the term $(a+s-\alpha)/s$ characterizes the back scattering of the packing. In light scattering experiments, the values of i^+ and i^- inside the medium are not accessible. Only the boundary values $i^+(L)$ (transmitted intensity) and $i^-(0)$ (back scattered intensity), where L is the thickness of the sample, can be obtained. Usually, values for the transmitted intensity are very low and difficult to measure due to the thickness of the sample [4]. Reducing the thickness allows direct transmission, which invalidates the diffusion approximation adopted. Therefore, computer simulations can be very useful when the basic mechanism of the radiative heat transfer is well defined, which is the case in geometric optics. Yang *et al.* [6] presented a Monte Carlo method to compute the transmitted intensity of the packings of spheres with different heights. An intrinsic problem of this method is the determination of the packing height, which has a deviation of plus or minus one sphere diameter. In the present study, a ray-tracing method is proposed to produce data on i^+ and i^- inside the medium, assuming that the medium is large enough, so that $i^-(L) \cong 0$. Directly accessing these values eliminates the difficulties relating to the packing height, as was the case in previous works.

MONTE CARLO METHOD AND RAY TRACING PROGRAM

The computer simulation presented here is based on a Monte Carlo method associated with a ray tracing program. As a ray tracing method is used, the limits of geometrical optics must be respected. Consider that the medium is composed by a packing of spherical particles with radius R. In this case, a size parameter can be defined as

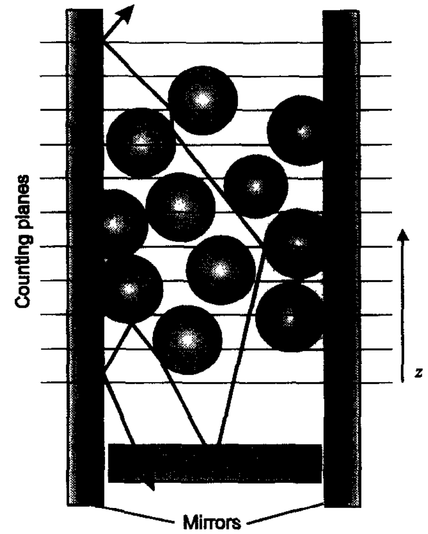

Fig. 2. Schematics of ray-tracing simulation for a packing of monosized spheres.

$$\beta = \frac{2\pi R}{\lambda} \quad (9)$$

where λ is the wavelength of the incident radiation. The lower limit for geometric optics, where there is no diffraction, is $\beta \approx 115$ [4]. This limit, applied to thermal (infrared) radiation, gives a minimum particle radius of approximately 75 μm. The particles are also assumed to be opaque, although refraction and internal reflections could easily be taken into account in the analysis.

The geometry of the system is described in Fig. 2. The flow is being analyzed in the z-direction, the forward direction being positive z. An emitting parallelogram, orthogonal to the z-axis, is placed below the packing. From this plane, a point and a vector with a positive z component are randomly chosen. They represent the origin and initial direction of a ray. A certain energy bundle is associated with this ray. The ray is traced into the packing and the closest ray-particle collision is determined. The normal to the surface of the particle at the collision point is calculated. The ray is then specularly reflected around the collision point, with the reflected angle being equal to the incident angle, and the reflected ray remaining in the plane defined by the surface normal and the incident ray. The energy bundle I associated with the ray before the collision is reduced to $(1-\varepsilon)I$, where ε is the particle surface emissivity. Counting planes orthogonal to the flow direction are placed at regular intervals through the packing. Each time a ray crosses one of these planes, the ray energy is added to the accumulated forward or backward energy of the plane, depending on whether the z component of the ray direction vector is positive or negative. Therefore, the first counting plane is used to determine the total incident intensity and the packing back scattered intensity. The packing is bounded by mirror like

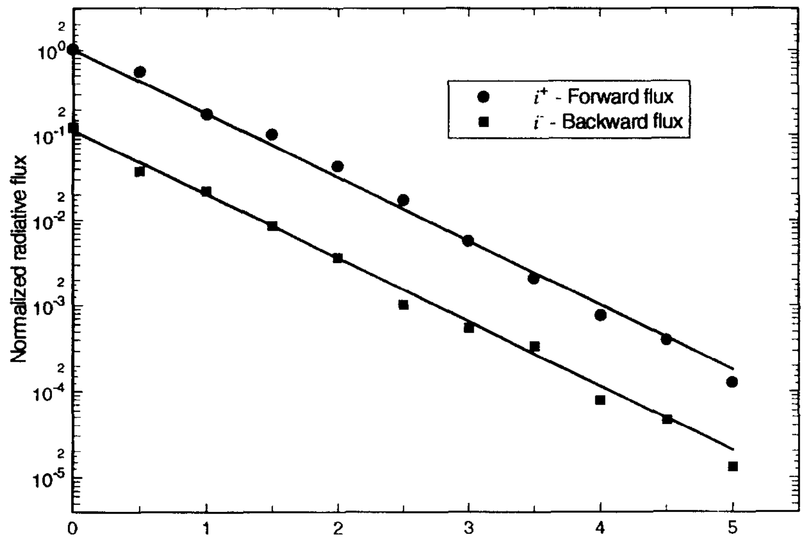

Fig. 3. Normalized forward and backward fluxes inside the medium.

planes parallel to the direction of the flow. These planes represent the reflecting symmetry condition, assuming that the packing can be repeated in the other directions. They specularly reflect the rays without changing their energy bundle. The planes actually cut the outer spheres of the packing to avoid ray defections over the boundaries. After a collision, the tracing continues until there is no significant ray energy left or the ray reaches the bottom or the top of the packing and escapes. The process is then repeated for a large number of rays (around 10 000 for the present study). Finally, the values of the forward and backward fluxes (i^+ and i^-) are collected from the counting planes, leading to a relation between the flux and the position in the flow direction.

APPLICATION TO A COMPUTER GENERATED RANDOM PACKING OF SPHERES

The procedure described above has been applied to a numerically generated random packing of spheres. This packing was obtained by Bouvard and Auvinet [9], with an algorithm proposed by Auvinet [10] that simulates the vertical deposition of the spheres inside a quasi-cubic box. This packing contains 420 rigid spheres in equilibrium under gravity. The average coordination number is six and the relative density is 0.57. To find out how the radiative exchange factor varies with the density of packing, the packing was densified using the homogeneous isotropic analogy proposed by Arzt [11]. Instead of decreasing the volume of the packing it was assumed that every particle expanded from its initial radius R to a radius R' with its center remaining fixed. Therefore, the new particles are truncated spheres. The new density is calculated using a random allocation method. A high number of coordinates are randomly chosen and their location, inside a particle or void, is ascertained. The final ratio of solid phase points to total points gives the relative density of the packing.

After collection of the flux from the counting planes, equations (8) are fitted using the least squares method. Figure 3 shows the flux obtained from the numerical simulation and the fitted relation for the initial packing with emissivity 0.4. Notice the good agreement between the simulation and the analytical relation. The discrete approach adopted reproduces well the continuous relation originally derived by Schuster. It is clear that a packing height of five diameters is sufficient to reduce the transmitted intensity by almost four decades, which eliminates the need to consider larger packings and validates the boundary conditions [equation (7)]. Figure 4 shows the radiative exchange factor obtained for different densities and emissivities. The exchange factor decays linearly with the density with an approximate slope of -1.1. The variation with the emissivity is also linear, the exchange factor increases with the emissivity with an approximate slope of 0.75. For the density and emissivity range studied, these variables seem to have independent action on the coefficient. These results are compared with the values obtained numerically by Yang *et al.* [6]. The experimental values obtained by Kasparek (Vortemeyer [12]) are also shown in this figure. Kasparek's experiment consisted in measuring the radiative exchange in a packing of metal spheres welded in layers. The layers were oriented perpendicularly to the direction of the flow. The experiment was performed in vacuum and the adjacent layers had no direct contact, therefore convection and conduction through the solid phase were eliminated. Notice the good agreement between the above mentioned experimental work and the present simulations. The agreement with Yang's study is not so good.

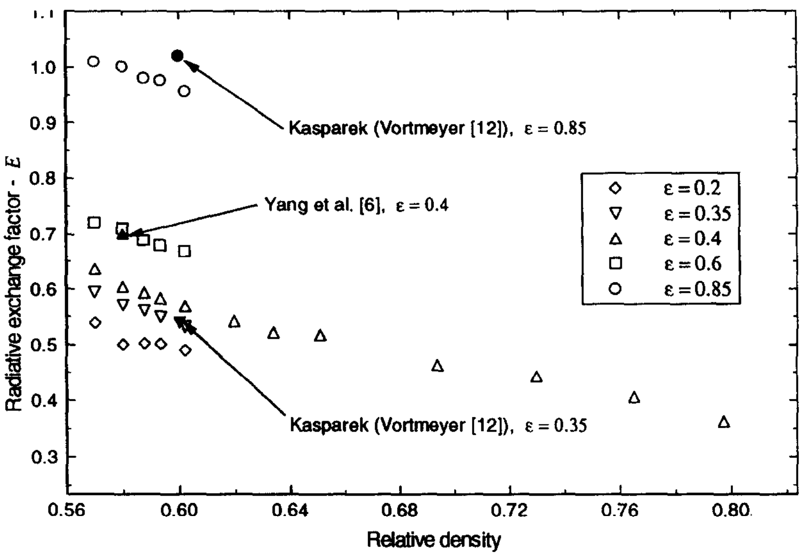

Fig. 4. Dimensionless radiative exchange factor as a function of relative density for different emissivities.

Figure 5 shows the absorption and scattering coefficients as a function of relative density for a particle emissivity $\varepsilon = 0.4$. The agreement with Yang's result for the absorption coefficient is very good, whereas a significant discrepancy is observed for the scattering coefficient. It is important to notice that these values reflect the geometric properties of the particulate medium under consideration. Since the media are not identical, some variations are normal and expected. Further investigations on the sensitivity of the coefficients to variations in the packing structure are necessary.

CONCLUSION

A new method to determine the radiative properties of porous media has been proposed. It is based on the Monte Carlo algorithm and a ray-tracing program. The radiative properties are determined from the forward and backward fluxes measured directly inside the medium. The ray tracing method described can be applied to a broad range of problems involving porous and particulate media. The only requirement is the applicability of geometrical optics, as defined by the size parameter [equation (9)]. It has been shown that, in the domain where geometric optics apply and where the surface properties of the particles and the structure of the packing are known, the numerical method proposed has clear advantages over traditional experimental and numerical methods. The radiative properties of other types of porous media could be easily studied using the present method. One example of such a medium is the packing of cylindrical rods analyzed by Vafai and Ettefagh [14].

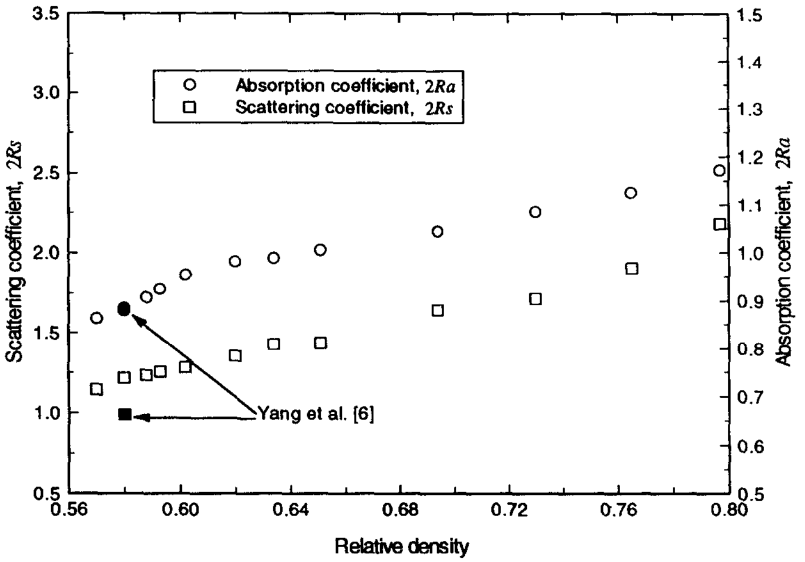

Fig. 5. Dimensionless absorption and scattering coefficients as a function of relative density for emissivity, $\varepsilon = 0.4$.

The dependence of E exchange factor for a random packing of spheres on its relative density and on the emissivity of particle surface has been found. As expected, E decreases with increasing density and increases with increasing emissivity.

The initial application of the present work was to exactly determine the radiative contribution to the effective conductivity of metal powders used in powder metallurgy. Although these powders are subjected to very high temperatures in vacuum, it has been shown that the radiative contribution was negligible compared to the solid phase conduction [3, 13]. However, the usefulness of the present work extends far beyond this particular application.

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